Efficiency requires innovation

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Abstract

In estimation a parameter $\theta \in \mathbb{R}$ from a sample $(x_1, \ldots, x_n)$ from a population P_θ a simple way of incorporating a new observation x_{n+1} into an estimator $\tilde{\theta}_n = \tilde{\theta}_n(x_1, \ldots, x_n)$ is transforming $\tilde{\theta}_n$ to what we call the *jackknife extension* $\tilde{\theta}^{(e)}_{n+1} = \tilde{\theta}^{(e)}_{n+1}(x_1, \ldots, x_n, x_{n+1})$,

$$\tilde{\theta}^{(e)}_{n+1} = \{\tilde{\theta}_n(x_1, \ldots, x_n) + \tilde{\theta}_n(x_{n+1}, x_2, \ldots, x_n) + \ldots + \tilde{\theta}_n(x_1, \ldots, x_{n-1}, x_{n+1})\}/(n+1).$$

Though $\tilde{\theta}^{(e)}_{n+1}$ lacks an innovation the statistician could expect from a larger data set, it is still better than $\tilde{\theta}_n$,

$$\mathrm{var}(\tilde{\theta}^{(e)}_{n+1}) \le \frac{n}{n+1} \mathrm{var}(\tilde{\theta}_n).$$

However, an estimator obtained by jackknife extension for all n is asymptotically efficient only for samples from exponential families. For a general P_θ, asymptotically efficient estimators require innovation when a new observation is added to the data.
Some examples illustrate the concept.